NJ-1310

B.Sc. (Part - II) Examination, Mar.-Apr., 2023

MATHEMATICS

Paper - II

(Differential Equations)

Time Allowed : Three Hours

Maximum Marks : 50

Minimum Pass Marks : 17

Note : Answer any two parts from each question. All questions carry equal marks.

UNIT-I

Q. 1. (a) Obtain series solution of the differential equation $xy'' + y' + xy = 0$.

(b) Express $f(x) = x^4 + 2x^3 + 2x^2 - x - 2$ in terms of Legendre polynomials.

P.T.O.

(c) Obtain the Eigen values and Eigen functions of the Sturm Liouville problem :

$$\frac{d^2y}{dx^2} + \lambda y = 0, \; y(0) = 0, \; y(\pi) = 0$$

UNIT-II

Q. 2. (a) Find $L^{-1}\{e^{-t}(2\sin h\,2t - 5\cos h\,2t)\}$.

(b) Evaluate :

$$L^{-1}\left\{\frac{1}{p^4} - \frac{3p}{p^2+16} + \frac{5}{p^2+4}\right\}$$

(c) Solve $(D^2 - D - 2)y = 20 \sin 2t$, $y(0) = -1$, $y'(0) = 2$.

UNIT-III

Q. 3. (a) Solve :

$$x(y^2 + z)p - y(x^2 + z)q = z(x^2 - y^2)$$

(b) Find the complete integral (C.I.) of :

$$p^3 + q^3 = 27z$$

(c) Solve by Charpit's method :

$$(p^2 + q^2)y = qz$$

UNIT-IV

Q. 4. (a) Classify and reduce the equation :

$$(n - 1)^2\, r - y^{2n}\, t - ny^{2n-1}\, q = 0$$

(b) Solve :

$$r - t = x - y$$

(c) Solve :

$$pt - qs = q^3$$

UNIT-V

Q. 5. (a) Find the distance between the curves $y = x$ and $y = x^2$ in the interval $[0, 1]$.

(b) Test for extremum of the functional :

$$I[y(x)] = \int_0^{\pi/2} (y'^2 - y^2)\,dx, \; y(0) = 0, \; y\left(\frac{\pi}{2}\right) = 1$$

(c) Find the function on which the following function can be extremized :

$$I[y(x)] = \int_0^1 (y'' - 2xy)dx,$$

$$y(0) = y'(0) = 0, \; y(1) = \frac{1}{120}$$

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